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Fundamental correlations for laminar and turbulent free convection from a uniformly heated vertical plate

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Abstract

A dimensionless parameter Π_Q is introduced for the correlation of the heat transfer in natural convection induced by a constant wall heat flux. This dimensionless parameter depends on the Prandtl number, *Pr*, and the modified Rayleigh number, Ra^*, as $\Pi_Q \sim Ra^*/(1+Pr^{-1})$. The development of Π_Q is based on physical arguments and allows the use of a single heat transfer correlation over the entire Prandtl number range. Dimensional arguments in terms of the laminar boundary layer and turbulent microscale concepts lead to a $Nu \sim \Pi_Q^{1/5}$ and $Nu \sim \Pi_Q^{1/4}$ dependence for the laminar and turbulent heat transfer, respectively. Comparison of these correlations with existing published data shows a good agreement.

1. Introduction

Natural convection from a vertical plate with constant wall heat flux, q_w, has been the subject of much research over the last few decades due to its relevance to a variety of industrial applications and naturally occurring processes, such as electronics cooling and solar heating. Numerous attempts have been made to correlate the heat transfer from the surface in terms of the appropriate dimensionless numbers. These correlations generally relate the Nusselt number, *Nu*, to the modified Rayleigh number, $Ra^* = g\beta q_w \ell^4/\nu\alpha k$, and Prandtl number, $Pr = \nu/\alpha$, with the asymptotic limits of $Nu = f(Ra^*)$ for $Pr \to \infty$ and $Nu = f(Ra^* Pr)$ for $Pr \to 0$. Though most experiments have focused on air and water (with $Pr \sim 1$–10), several attempts have been made over the years to develop generalized natural convection correlations that would apply to a range of Prandtl numbers. Table 1 summarizes some of these correlations for the laminar and turbulent cases. We will mention a few here.

One of the earliest attempts in the laminar case was that of Sparrow and Gregg [1], who presented an exact similarity solution of the laminar boundary layer equations for Prandtl numbers of 0.1, 1, 10 and 100 and suggested

$$Nu = 0.616\left(\frac{Pr^2}{0.8+Pr}\right)^{1/5} Gr^{*1/5}, \tag{1}$$

where $Gr^* = Ra^*/Pr$ is the modified Grashof number. This correlation was recently validated in an experimental study by Pitman et al. [2] and agreed well in the range $10^2 \leqslant Gr^* \leqslant 10^7$ with the experimental results of Goldstein and Eckert [3] for water. Other attempts at correlating a broad range of *Pr* numbers include the work of Churchill and Ozoe [4] and that of Fujii and Fujii [5], which agreed well with the theoretical work of Chen et al. [6] on laminar free convection in boundary layer flows from horizontal, inclined, and vertical flat plates involving air and water. Other investigators focused on moderate Prandtl number fluids only. Qureshi and Gebhart [7] developed a correlation for water at room temperature ($Pr = 6$), which agrees well with the more recent data obtained by King and Reible [8]. For low Prandtl numbers we note the solution of Chang et al. [9] using perturbation techniques. Mercury was also used in a number of experiments, notably the work

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Nomenclature

c	specific heat
C_0, C_1	constants
g	gravitational acceleration
h	convection heat transfer coefficient
$\mathscr{K}$	mean kinetic energy of velocity fluctuations
$\mathscr{K}_\theta$	root mean square of temperature fluctuations
k	thermal conductivity
ℓ	characteristic length
Gr^*	modified Grashof number, $Gr^* = Ra^*/Pr$
Nu	Nusselt number, $Nu = h\ell/k$
$\mathscr{P}$	inertial production
$\mathscr{P}_N$	buoyant production
$\mathscr{P}_\theta$	thermal production
Pr	Prandtl number, $Pr = \nu/\alpha$
q_w	imposed wall heat flux
Ra	Rayleigh number, $Ra = g\beta\Delta T\ell^3/\nu\alpha$
Ra^*	modified Rayleigh number, $Ra^* = g\beta q_w \ell^4/\nu\alpha k$
T	temperature
u, U	turbulent velocity scale
v, V	dissipation scale velocity; laminar velocity

Greek symbols

α	thermal diffusivity
β	coefficient of volumetric thermal expansion
Δ	difference
δ	boundary layer thickness
ϵ	viscous dissipation
ϵ_θ	thermal dissipation
η_θ	thermal Kolmogorov scale
θ, Θ	temperature
λ	Taylor scale
μ	dynamic viscosity
ν	kinematic viscosity
Π_N	natural convection dimensionless number
Π_Q	natural convection dimensionless number for constant heat flux
ρ	density

Subscripts

θ	thermal
w	at wall

Table 1
Laminar and turbulent correlations

Author	Correlation
Laminar correlations	
Sparrow and Gregg [1]	$Nu = 0.616[Pr^2/(0.8 + Pr)]^{1/5} Gr^{*1/5}$
Goldstein and Eckert [3]	$Nu = 0.586 Gr^* Pr^{1/5}$, $10^2 \leqslant Gr^* \leqslant 10^7$, water
Chang et al. [9]	$Nu = 0.632 Pr^{0.37} Gr^{*1/5}$, low Pr
Julian and Akins [10]	$Nu = 0.196 Gr^{*0.188}$, $10^4 \leqslant Gr^* \leqslant 10^9$, mercury
Humphreys and Welty [11]	$Nu = 0.196 Gr^{*0.188}$, $10^6 \leqslant Gr^* \leqslant 10^{11}$, mercury
Colwell and Welty [12]	$Nu = 0.23 Gr^{*0.18}$, $10^4 \leqslant Gr^* \leqslant 10^9$, mercury
Churchill and Ozoe [4]	$Nu = 0.726 Ra^{*1/5}/[1 + (0.437/Pr)^{9/16}]^{16/45}$
Fujii and Fujii [5]	$Nu = [Pr/(4 + 9Pr^{1/2} + 10Pr)]^{1/5} Gr^* Pr^{1/5}$
Qureshi and Gebhart [7]	$Nu = 0.587 Ra^{*1/5}$, $1.2 \times 10^6 \leqslant Ra^* \leqslant 1.2 \times 10^{13}$, water
Martynenko et al. [13]	$Nu/Ra^* Pr^{1/5} = f(Ra^* Pr^{-1/5})$, low Pr
Turbulent correlations	
Vliet and Liu [16]	$Nu = 0.568 Gr^* Pr^{0.22}$, $2 \times 10^{13} \leqslant Gr^* \leqslant 10^{16}$, water
Miyamoto et al. [18]	$Nu = 0.104 Ra^{*0.272}$, $1.5 \times 10^{13} \leqslant Gr^* \leqslant 1.7 \times 10^{14}$, air
Vitharana and Lykoudis [19]	$Nu = 0.064 Ra^* Pr^{1/3}$, mercury

of Julian and Akins [10], Humphreys and Welty [11], and Colwell and Welty [12]. More recently, Martynenko et al. [13] obtained a correlation for low Prandtl numbers, which was in agreement with the experimental results of Ede [14], Julian and Akins [10] and Chang and Akins [15].

Less results are available in the existing literature for the turbulent case. Vliet and Liu [16] proposed an empirical correlation for turbulent flow based on their experimental investigation of natural convection along a uniformly heated vertical flat plate using water as the working fluid. The experimental measurements of Qureshi and Gebhart [7] and Inagaki and Komori [17] correlated with their results. Turbulent free convection was investigated for air by Miyamoto et al. [18] and for mercury by Vitharana and Lykoudis [19].

The correlations mentioned above all show a $Nu = f(Ra^*, Pr)$ dependence; however, the form of the correlation is not uniform across the studies. A similar situation exists in the case of natural convection driven by an imposed temperature difference, ΔT (e.g., hot isothermal plate). There, the independent dimensionless numbers characterizing the natural convection are the usual Rayleigh and Prandtl numbers, $Ra = g\beta\Delta T\ell^3/\nu\alpha$

and Pr, respectively. Using arguments based on the coupling of the momentum and energy equations, a fundamental dimensionless parameter, $\Pi_N \sim Ra/(1 + Pr^{-1})$, was recently proposed by Arpacı [20,21] for the characterization of the heat transfer in buoyancy-driven flows, i.e., $Nu = f(\Pi_N)$. This parameter appeared previously in several correlations, such as the boundary layer solution of Squire [22] and the experimental results of Catton [23]. However, rather than being a purely empirical parameter, the existence of Π_N as a dimensionless number characterizing buoyancy-driven flows was derived from physical arguments. The main objectives of the current study are to extend this development to the case of natural convection along a vertical plate with constant heat flux and develop heat transfer correlations that would apply to all fluids for laminar and turbulent flow in terms of the appropriate fundamental dimensionless number, Π_Q. In Section 2, dimensional arguments lead to the development of Π_Q, followed by a discussion of laminar and turbulent heat transfer in terms of Π_Q. In Section 3 results from new correlations in terms of Π_Q are introduced and discussed. Section 4 concludes the study.

2. Dimensional analysis

2.1. A dimensionless number

Starting from the premise that the appropriate, physically meaningful dimensionless number that characterizes the heat transfer in natural convection flows must result from arguments based on *both* the conservation of momentum and thermal energy, Arpacı [21] proposed a dimensionless number which explicitly describes natural convection driven by a temperature difference, ΔT, in terms of $Ra = g\beta\Delta T\ell^3/\nu\alpha$ and $Pr = \nu/\alpha$. In the development, dimensional arguments were used to express the dimensionless momentum equation in terms of characteristic quantities, and to eliminate velocity, which is a dependent variable in natural convection, using the thermal energy balance. The resulting fundamental dimensionless number, $\Pi_N \sim Ra/(1 + Pr^{-1})$, was shown to correlate the heat transfer in buoyancy-driven flows. The numeral 1 implies an order of magnitude; it is explicitly replaced by a coefficient C_0, when the various force balance ratios are converted to equalities.

In the case of an imposed heat flux, q_w, the temperature difference driving the motion, ΔT, must be expressed in terms of q_w. Consider a control volume parallel to the vertical plate and let ℓ be a characteristic length and k the thermal conductivity of the fluid. ΔT may be obtained from the energy balance at the wall, $q_w \sim k\Delta T/\ell$, which may be substituted into the dimensionless parameter obtained by Arpacı [21] to yield a generalized dimensionless number composed of independent variables only,

$$\Pi_Q \sim \frac{(g\beta q_w\ell^3/\nu k)(\ell/\alpha)}{1 + (\ell/\nu)(\alpha/\ell)} \sim \frac{g\beta q_w\ell^4/\nu\alpha k}{1 + \alpha/\nu}, \tag{2}$$

where the numeral 1 implies an order of magnitude. Eq. (2) may be rewritten in terms of Pr and $Ra^* = g\beta q_w\ell^4/\nu\alpha k$ as

$$\Pi_Q \sim \frac{Ra^*}{1 + Pr^{-1}} \sim \left(\frac{Pr}{1 + Pr}\right)Ra^*, \tag{3}$$

where Π_Q is the appropriate dimensionless number for natural convection with an imposed heat flux in any fluid. This parameter converges towards the proper Rayleigh–Prandtl combinations in the limits of low and high Prandtl numbers

$$\lim_{Pr\to\infty} \Pi_Q \to Ra^*, \qquad \lim_{Pr\to 0} \Pi_Q \to Ra^* Pr, \tag{4}$$

consistent with the heat transfer dependence in these limits. Accordingly, a more explicit relation for the heat transfer in natural convection driven by an imposed heat flux, q_w, is $Nu = f(\Pi_Q)$ for any fluid. This can bridge the gap between low- and high-Prandtl-number correlations using a single dimensionless number. Explicitly,

$$\Pi_Q = \frac{Ra^*}{1 + C_0 Pr^{-1}} = \frac{Ra^* Pr}{C_0 + Pr}, \tag{5}$$

where the coefficient C_0 depends on the flow structure and results from converting the dimensionless force and energy balances used to develop Eq. (2) to equalities. Although the existence of Π_Q has never been previously directly shown, the similarity solution, Eq. (1), developed by Sparrow and Gregg [1] more than four decades ago, and recently validated experimentally [2], leads after replacing Gr^* by Ra^*/Pr to an expression in terms of Π_Q,

$$Nu = 0.616\left(\frac{Pr^2}{0.8 + Pr}\right)^{1/5} Gr^{*1/5} = 0.616\left(\frac{Ra^* Pr}{0.8 + Pr}\right)^{1/5}. \tag{6}$$

Here we have used physical arguments to show the existence of Π_Q. Next, we extend these arguments to laminar convection.

2.2. Laminar convection

The following intuitive considerations in terms of the usual laminar boundary layer concepts illustrate the use of Π_Q to correlate laminar heat transfer. Consider natural convection from a vertical plate subject to a constant heat flux, q_w. On dimensional grounds the momentum balance, integrated over the momentum boundary layer thickness, δ, may be written as

$$g\frac{\Delta\rho}{\rho} \sim V\frac{V}{\ell} + \nu\frac{V}{\delta^2}, \tag{7}$$

showing the balance between the (driving) buoyancy force and the inertial and viscous forces, per unit mass. As usual, the viscous diffusion is assumed to be confined to the viscous boundary layer. Here V is a characteristic velocity scale, ℓ an integral (geometric) scale, and $\Delta\rho/\rho \sim \beta\Delta T$ the heat flux induced density gradient driving the buoyancy. The characteristic velocity, $V \sim \alpha\ell/\delta_\theta^2$, can be obtained by considering the balance between enthalpy flow and conduction, where the heat diffusion (conduction) is again confined to the thermal boundary layer of thickness δ_θ. Similarly, the energy balance at the wall,

$$q_w \sim k\frac{\Delta T}{\delta_\theta}, \tag{8}$$

yields the induced temperature difference ΔT. Following the Squire [22] postulate for buoyancy-driven flows, assume that $\delta \sim \delta_\theta$ in Eq. (7). Rather than suggesting the equality of these two boundary layer thicknesses, this assumption postulates the secondary importance of the difference between δ and δ_θ for the heat transfer. This postulate has been well tested in natural convection, even in cases where δ/δ_θ differs considerably from unity. Using this assumption and substituting the expressions for V and ΔT, the momentum balance may be rearranged as

$$\frac{\ell}{\delta_\theta^5}\left(1+\frac{\alpha}{\nu}\right) \sim \frac{g\beta q_w}{\nu\alpha k}, \tag{9}$$

or, in terms of Π_Q,

$$\frac{\delta_\theta}{\ell} \sim \left(1+\frac{1}{Pr}\right)^{1/5} Ra^{*-1/5} \sim \Pi_Q^{-1/5}. \tag{10}$$

Recalling the definition of the heat transfer coefficient, h, at an interface, the heat transfer balance at the wall may be expressed as $k\Delta T/\delta_\theta \sim h\Delta T$. The Nusselt number may thus be expressed in terms of the integral and thermal boundary layer scales as

$$Nu = \frac{h\ell}{k} \sim \frac{\ell}{\delta_\theta} \sim \Pi_Q^{1/5}. \tag{11}$$

We thus obtain a general form of the Nusselt number for laminar natural convection driven by an imposed heat flux in terms of Π_Q. This equation provides the correct classical limits $Nu \sim Ra^{*1/5}$ and $Nu \sim (Ra^* Pr)^{1/5}$ for $Pr \to \infty$ and $Pr \to 0$, respectively. Investigation of this result in terms of experimental data will be done in Section 3. Next, we extend the preceding development to the turbulent case.

2.3. Turbulent convection

The development that follows uses the microscale concepts first introduced by Arpacı for buoyancy-driven flows and extends them to flows induced by a constant wall heat flux. The original development is briefly reviewed first.

Following the usual approach, we decompose the instantaneous turbulent quantities into a temporal mean (denoted by capital letters) and fluctuations (denoted by lower-case letters), $\tilde{u}_i = U_i + u_i$ and $\tilde{\theta} = \Theta + \theta$, and assume the mean quantities to be statistically steady. For a homogeneous pure shear flow, the Reynolds averaged equations for the mean kinetic energy of the velocity fluctuations, $\mathscr{K} = 1/2\overline{u_i u_i}$, reduce to

$$\mathscr{P}_N = \mathscr{P} + (-\epsilon). \tag{12}$$

Here $\mathscr{P}_N$ in Eq. (12) is the buoyant energy production, $\mathscr{P} = -\overline{u_i u_j}S_{ij}$ is the inertial production, and $\epsilon = 2\nu\overline{s_{ij}s_{ij}}$ is the viscous dissipation of turbulent kinetic energy, where S_{ij} and s_{ij} are the mean and fluctuating strain rates, respectively. Eq. (12) states that the buoyant turbulent kinetic production is partly converted into inertial production and partly into viscous dissipation. Similarly, the balance for the root mean square of the temperature fluctuations, $\mathscr{K}_\theta = 1/2\overline{\theta^2}$, reduces to

$$\mathscr{P}_\theta = \epsilon_\theta, \tag{13}$$

where $\mathscr{P}_\theta = -\overline{u_j\theta}(\partial\Theta/\partial x_j)$ and $\epsilon_\theta = \alpha\overline{(\partial\theta/\partial x_j)(\partial\theta/\partial x_j)}$ represent the thermal production and the thermal dissipation, respectively.

On dimensional grounds, assuming $S_{ij} \sim u/\ell$ and $\partial\Theta/\partial x_j \sim \theta/\ell$, Eqs. (12) and (13) may be written as

$$\mathscr{P}_N \sim \frac{u^3}{\ell} + \nu\frac{u^2}{\lambda^2}, \tag{14}$$

$$u\frac{\theta^2}{\ell} \sim \alpha\frac{\theta^2}{\lambda_\theta^2}, \tag{15}$$

where u and θ respectively denote the rms values of velocity and temperature fluctuations, ℓ is an integral scale, and λ and λ_θ are the kinetic and thermal Taylor scales, respectively. Invoking the Squire [22] postulate in Eq. (14) results in a thermal Taylor scale [21]

$$\lambda_\theta \sim \ell^{1/3}\left(1+\frac{1}{Pr}\right)^{1/6}\left(\frac{\nu\alpha^2}{\mathscr{P}_N}\right)^{1/6} = \ell^{1/3}(1+Pr)^{1/6}\left(\frac{\alpha^3}{\mathscr{P}_N}\right)^{1/6}, \tag{16}$$

appropriate for natural convection. If instead we reinterpret Eq. (14) in terms of the dissipation scales η, η_θ and v, we can deduce the thermal Kolmogorov scale [21] for natural convection flows as

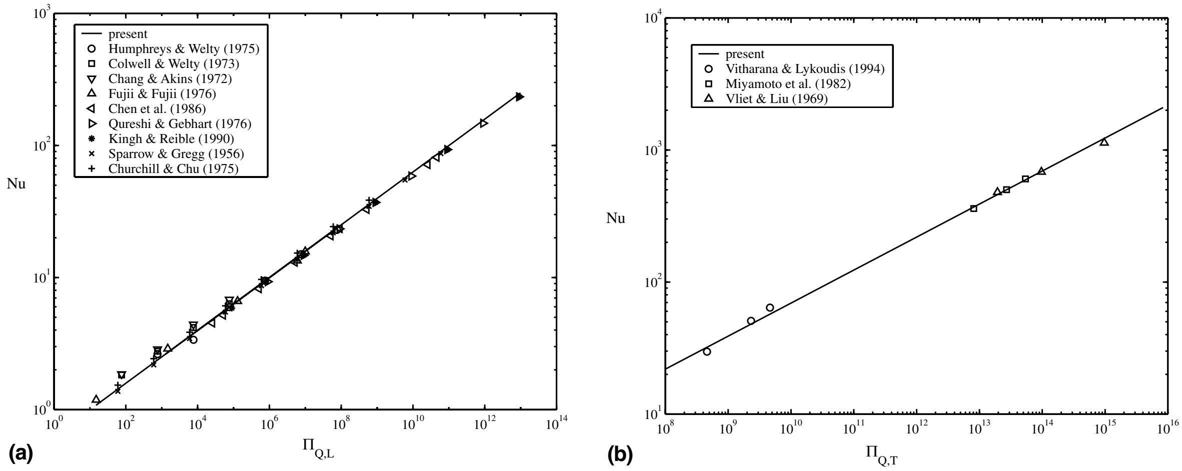

Fig. 1. Variation of Nu with (a) $\Pi_{Q,\mathrm{L}}$ for the laminar case and (b) $\Pi_{Q,\mathrm{T}}$ for the turbulent case.

$$\eta_\theta \sim \left(1+\frac{1}{Pr}\right)^{1/4}\left(\frac{\nu\alpha^2}{\mathscr{P}_N}\right)^{1/4} = (1+Pr)^{1/4}\left(\frac{\alpha^3}{\mathscr{P}_N}\right)^{1/4}, \tag{17}$$

which can be shown to recover the classical Batchelor, Oboukhov–Corrsin and Kolmogorov scales in the limits of $Pr \to \infty$, 0, and 1, respectively [21]. Given that $\mathscr{P}_N \sim g\beta v\theta$ depends on velocity, Eqs. (16) and (17) expressed in terms of velocity cannot be the ultimate forms of the Taylor and Kolmogorov scales. An estimate of the thermal velocity scale, v, may be obtained from Eq. (15) expressed in terms of dissipation scales, while the temperature scale, θ, may be obtained by considering an energy balance in a layer of thickness η_θ, i.e.,

$$v \sim \frac{\alpha}{\eta_\theta} \quad \text{and} \quad q_\mathrm{w} \sim k\frac{\theta}{\eta_\theta}. \tag{18}$$

Substituting Eq. (18) into Eq. (17) yields the thermal Kolmogorov scale for natural convection induced by an imposed heat flux,

$$\eta_\theta \sim \left(1+\frac{1}{Pr}\right)^{1/4}\left(\frac{\nu\alpha k}{g\beta q_\mathrm{w}}\right)^{1/4} = (1+Pr)^{1/4}\left(\frac{\alpha^2 k}{g\beta q_\mathrm{w}}\right)^{1/4}. \tag{19}$$

Here again, the numeral 1 indicates an order of magnitude. As in the laminar case, the turbulent Nusselt number may be related to the ratio of the thermal Kolmogorov and integral length scales as $Nu \sim \ell/\eta_\theta$. Eq. (19) may then be rearranged to provide a general form of the Nusselt number for turbulent natural convection driven by an imposed heat flux in terms of Π_Q,

$$Nu \sim \frac{\ell}{\eta_\theta} \sim \Pi_Q^{1/4}. \tag{20}$$

3. Results and discussion

In the previous section, dimensional arguments were used to introduce a fundamental dimensionless number,

$$\Pi_Q = \frac{Ra^* Pr}{C_0 + Pr}, \tag{21}$$

appropriate for correlating natural convection about a vertical plate subject to a constant heat flux. The heat transfer may be correlated as

$$Nu = C_1\left(\frac{Ra^* Pr}{C_0 + Pr}\right)^n, \tag{22}$$

where $n = 1/5$ for laminar convection and $n = 1/4$ for turbulent convection. Although the values of C_0 and C_1 must be determined from experimental data, they are expected to be numerical constants for each case. Using the experimental and numerical data existing in the literature, the coefficients C_0 and C_1 were determined using a general least squares procedure and are:

$$\begin{aligned} &\text{Laminar:} && C_0 = 0.670, && C_1 = 0.630, && n = 1/5,\\ &\text{Turbulent:} && C_0 = 0.191, && C_1 = 0.219, && n = 1/4. \end{aligned} \tag{23}$$

Fig. 1(a) plots the Nusselt number, Nu, versus Π_Q in the laminar regime. Here Π_Q is defined as

$$\Pi_{Q,\mathrm{L}} = \frac{Ra^* Pr}{0.670 + Pr} \qquad \text{(laminar)}. \tag{24}$$

The solid line represents the correlation given by Eq. (22) with the appropriate values listed in Eq. (23). The symbols represent laminar experimental and theoretical results taken from the literature. These data covered a Prandtl number range of $0.001 \leqslant Pr \leqslant 1000$, i.e., a factor of 10^6 in the Prandtl domain. As can be seen from Fig. 1(a), the data agree well with the new correlation proposed and the parameter Π_Q can indeed be used to correlate the data. When heat transfer (Nu) values were not explicitly listed by the authors of the experimental studies, data points were calculated based on the empirical correlations given by the authors. For this reason, no attempt is made here to show the deviation of the proposed correlations from the data. However, a closer look at Fig. 1(a) indicates that these deviations are minor and fall within the realm of experimental uncertainty in measuring or computing the Nusselt number.

Fig. 1(b) plots the Nusselt number, Nu, versus Π_Q in the turbulent regime. Here Π_Q is defined as

$$\Pi_{Q,\mathrm{T}} = \frac{Ra^* Pr}{0.191 + Pr} \qquad \text{(turbulent)}. \tag{25}$$

The solid line represents the correlation given by Eq. (22) with the appropriate values listed in Eq. (23). Good agreement can be seen between the proposed correlation and the experimental data. The different values of the constant C_0 in Eqs. (24) and (25) indicate the lesser importance, from the standpoint of heat transfer, of inertial effects in turbulent flows than in laminar flows. In general, the value of C_0 is expected to be closer to one in the laminar case and an order of magnitude less in the turbulent case. Unlike the laminar case, there is little data in the literature for the turbulent case. Therefore, new experimental results for both low- and high-Prandtl-number fluids may enhance the correlation proposed for the turbulent region by yielding slightly more accurate values of the C_0 and C_1 coefficients.

4. Conclusions

A dimensionless number, $\Pi_Q \sim Ra^*/(1 + Pr^{-1})$, was introduced as a fundamental dimensionless parameter for the correlation of heat transfer in natural convection flows driven by an imposed heat flux. Heat transfer correlations of the form $Nu \sim \Pi_Q^{1/5}$ and $Nu \sim \Pi_Q^{1/4}$, which apply to all fluids, were predicted for laminar and turbulent convection, respectively, using boundary layer and microscale arguments. Comparison of the proposed correlations with existing experimental data showed good agreement.

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